ABBG (2010): Random Planted Model

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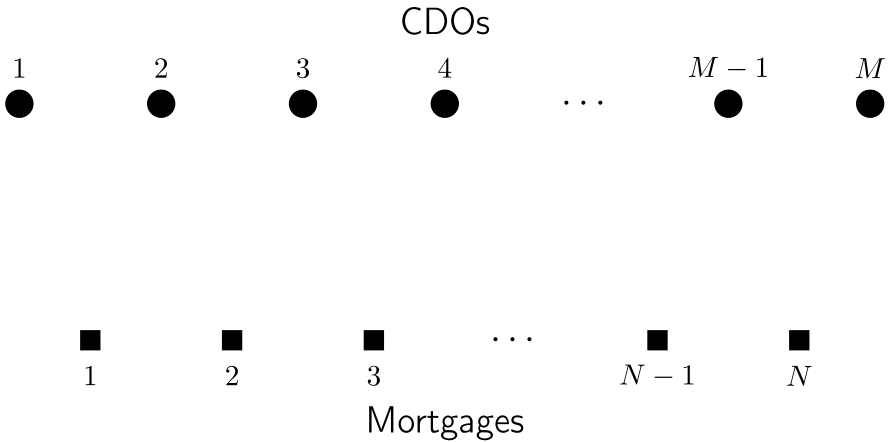
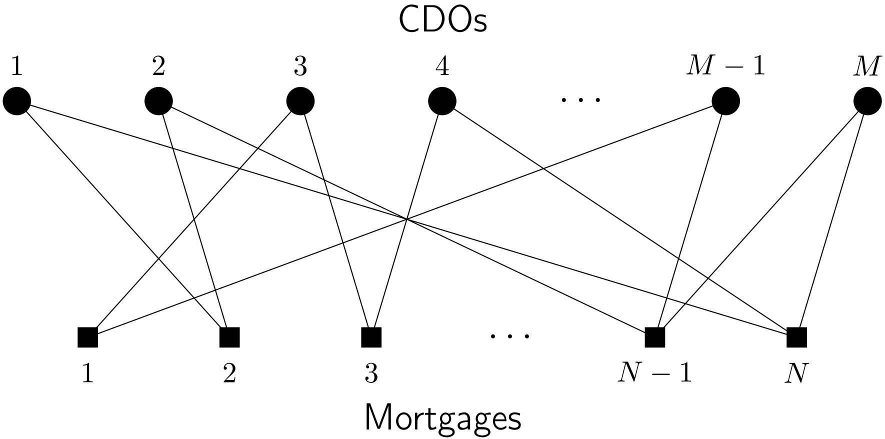
Goal: Create M CDOs containing $D = 2$ out of N mtgs.



Note: Each mortgage can be assigned to more than one CDO.

1. Framework
2. Random Graph
3. Planted Clique
4. Main Result

Random Graph: $g \sim \mathrm{F}_1(M, N, D)$



Step 1: For each CDO, randomly sample $D = 2$ mortgages.

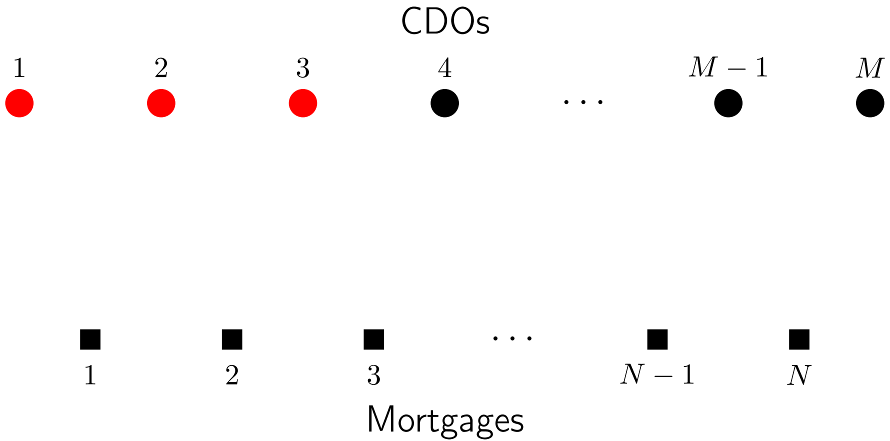
1 Framework

2 Random Graph

3 Planted Clique

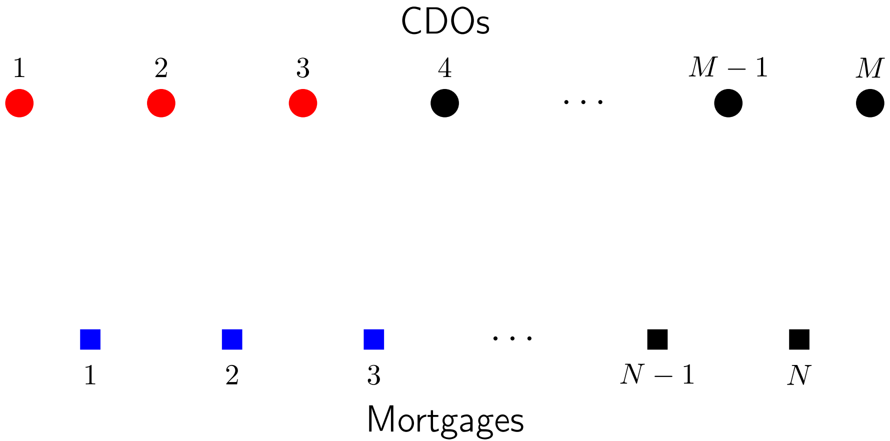
4 Main Result

Planted Clique: $g \sim \mathrm{F}_2(M, N, D; m, n, d)$



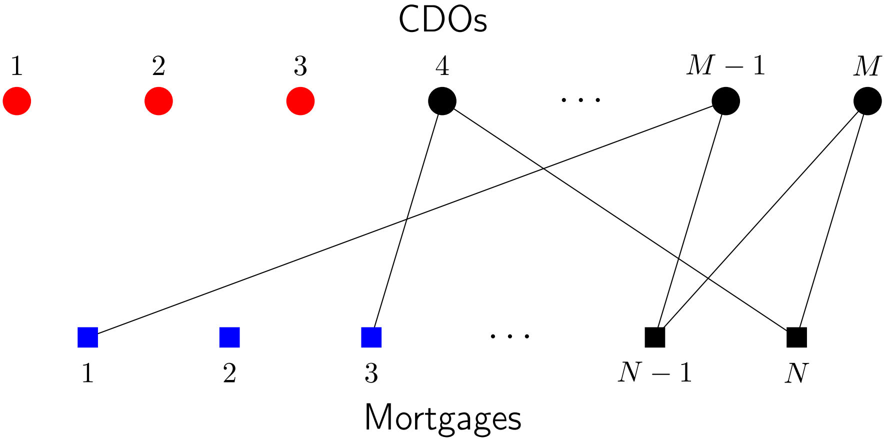
Step 1: Choose $m = 3$ CDOs to be in set $\mathcal{M}^*$ (red).

Planted Clique: $g \sim \mathrm{F}_2(M, N, D; m, n, d)$



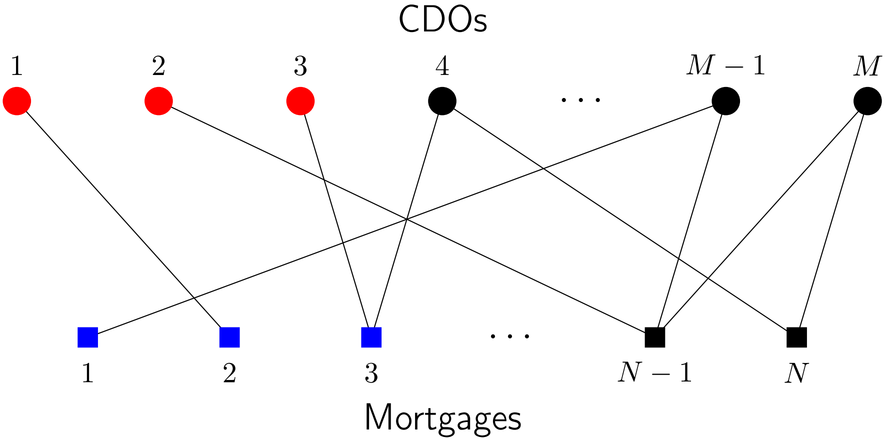
Step 2: Choose $n = 3$ mortgages to be in set $\mathcal{N}^*$ (blue).

Planted Clique: $g \sim \mathrm{F}_2(M, N, D; m, n, d)$



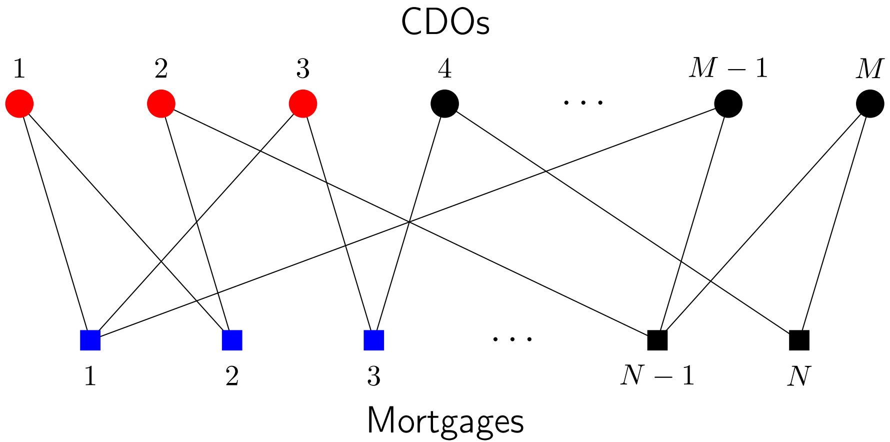
Step 3: $\forall\ m \notin \mathcal{M}^*$ randomly select $D = 2$ neighbors from $\mathcal{N}$.

Planted Clique: $g \sim \mathrm{F}_2(M, N, D; m, n, d)$



Step 4: $\forall\ m \in \mathcal{M}^*$ randomly select $D - d = 1$ neighbor from $\mathcal{N}$.

Planted Clique: $g \sim \mathrm{F}_2(M, N, D; m, n, d)$



Step 5: $\forall\ m \in \mathcal{M}^*$ randomly select $D - d = 1$ neighbor from $\mathcal{N}^*$.

1 Framework

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Constraints

$c1)$ $M \cdot D \geq N$: Each mtg should be related to at least one CDO.

$c2)$ $d \gg D \cdot (n/N)$: If not, then a random CDO will be expected to contain more mtgs from the collection of lemons $\mathcal{N}^*$ than a booby trapped CDO.

$c3)$ $m \cdot d \gg n$: If not, a random graph will be expected to have a graph as dense as the planted dense subgraph.

Random Planted Problem

Proposition

Suppose that:

$a1)$ $N = \mathrm{O}(M \cdot D)$
$a2)$ $(m \cdot d^2/n)^2 = \mathrm{o}(M \cdot D^2/N)$

Then there is no polynomial time algorithm to distinguish whether or not a g was drawn from F_1 or F_2.[a][b]

[a] $f = \mathrm{O}(x)$ if there exists a constant $M > 0$ s.t. $f(x) \leq M \cdot x$ as $x \to \infty$.
[b] $f = \mathrm{o}(x)$ if for any constant $M > 0$ we have that $f(x) \leq M \cdot x$ as $x \to \infty$.

$a1)$ $\Rightarrow$ each mtg is likely securitized more than once:

$$M \cdot D = \text{Gross Securitization Volume (\# Mtgs)}$$

$a2)$ $\Rightarrow$ lemons aren't planted too frequently relative to regular mtgs:

$$D \cdot \left(\frac{M \cdot D}{N}\right) = \mathrm{E}[\text{\# Times Securitized}], \quad d \cdot \left(\frac{m \cdot d}{n}\right) = \mathrm{E}[\text{\# Times Planted}]$$

Random Planted Problem

Proof (Intuition).

Strategy: Look for pairs of CDOs with too many shared mtgs.

Define the co-degree of vertices m and m'—written as $\mathrm{cod}(m, m')$—to be the number of common neighbors of m and m'.

If $D^2/N \gg 1$, then the LLN implies that in a random graph:

$$\lim_{M \cdot D \to \infty} \mathrm{cod}(m, m') \sim \mathrm{N}\left(D^2/N, \sqrt{D^2/N}\right)$$

In a dense subgraph, m and m' will have d^2/n more neighbors than otherwise, but...

...need to look through all possible *pairs* of CDOs!!! ...and hope that $\sqrt{D^2/N}$ is sufficiently small that some CDOs don't share a bunch of mtgs by chance!!!

□